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ELEMENTS OF ASTROMECHANICS

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This series of articles presents, in an elementary way, aspects of the following topics:

I. The kinematics of plane curvilinear motion and the geometry of conic sections.
II. The laws of Kepler and Newton's law of gravitation.
III. The two-body problem; orbital velocity, size and shape of orbits.
IV. Keplerian motion. The time element in orbits.
V. Energy relations in orbits. Elements of space flight.
VI. Analysis of double star orbits. Stellar masses and unseen companions.

I. The Kinematics of Plane Curvilinear Motion and the Geometry of Conic Sections.

1. *Velocity and acceleration in polar co-ordinates*

Astromechanics makes use of fundamental notions of kinematics. We shall restrict ourselves to studying plane curvilinear motion, referred to a Galilean or *inertial frame*. In an inertial reference frame the *law of inertia* holds; i.e., within reasonably large intervals of space and time and within the limits of observational resolving power, an object sufficiently far away from other objects does not suffer an acceleration.

For the case of plane curvilinear motion, the instantaneous velocity may be measured in three ways:

angular velocity

$$\omega = \frac{d\theta}{dt}, \tag{1.1}$$

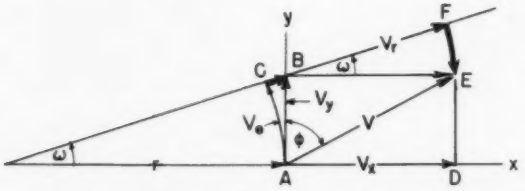

FIG. 1.1—Centrifugal and Coriolis accelerations in polar co-ordinates.

linear velocity

$$AE = V = \frac{r}{\cos\phi}\frac{d\theta}{dt}, \tag{1.2}$$

areal velocity

$$A = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{r\,V\cos\phi}{2}. \tag{1.3}$$

Here ϕ is the angle between the linear velocity and its component perpendicular to the radius vector; or, what amounts to the same, the angle between the normal to the velocity and the radius vector. The linear velocity components along and perpendicular to the radius vector are

$$V_r = \frac{dr}{dt} \qquad (\theta \text{ constant}), \tag{1.4}$$

$$V_\theta = r\frac{d\theta}{dt} = \frac{2A}{r} \qquad (r \text{ constant}). \tag{1.5}$$

The instantaneous acceleration components along and perpendicular to the radius vector *in the inherently rotating co-ordinate system* are:

$$\frac{dV_r}{dt} = \frac{d^2r}{dt^2} \qquad (\theta \text{ constant}), \tag{1.6}$$

$$\frac{dV_\theta}{dt} = r\frac{d^2\theta}{dt^2} \qquad (r \text{ constant}). \tag{1.7}$$

2. *Centrifugal and Coriolis accelerations*

Referred to an inertial frame of reference, observations made in the inherently rotating co-ordinate system are affected by a "falling outward" along the radius vector and a "falling behind" perpendicular to the radius vector, in addition to any true acceleration in the motion.

In the inherently rotating polar co-ordinate system, the instantaneous velocity components V_r and V_θ *rotate* continuously—the instantaneous

angular velocity being ω per unit time—with respect to initial values $V_r = V_x(AD)$ and $V_\theta = V_y(AB)$, which remain constant in the inertial (x,y) system. Over a sufficiently small unit of time, the scalar values of V_r and V_θ may be taken as constant, while their vectoral values change because of the rotation. Per unit time the arc AC of length V_θ is covered by the component velocity vector V_θ, and the segment BF of length V_r by the component velocity vector V_r.

Hence apart from any true acceleration, as a result of the angular rotation ω, the object has "fallen outward" as measured along the radius vector, the amount being

$$CB = (1 - \cos\omega)r = \frac{\omega^2}{2}r = V_\theta \cdot \frac{\omega}{2}. \tag{1.8}$$

Also the object has "fallen behind" as measured perpendicular to the radius vector, the amount being

$$FE = -V_r \cdot \omega \tag{1.9}$$

Over a sufficiently small unit of time, the acceleration may be considered uniform and hence the amount of falling is half the acceleration. Therefore, the amounts (1.8) and (1.9) correspond to the so-called

centrifugal acceleration

$$V_\theta \cdot \omega = r\left(\frac{d\theta}{dt}\right)^2, \tag{1.10}$$

and

Coriolis acceleration

$$-2V_r \cdot \omega = -2\frac{dr}{dt}\frac{d\theta}{dt}. \tag{1.11}$$

3. *Acceleration in polar co-ordinates referred to inertial frame*

Any accelerations (1.6) and (1.7) along and perpendicular to the radius vector must be corrected for the accelerations (1.10) and (1.11) inherent in the rotating co-ordinate system to obtain the accelerations f_r and f_θ in plane curvilinear motion referred to an inertial system, but measured in the rotating polar co-ordinate system:

$$f_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2, \tag{1.12}$$

$$f_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}. \tag{1.13}$$

The latter formula may also be written as

$$f_\theta = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = \frac{2}{r}\frac{dA}{dt}. \tag{1.14}$$

(*a*) Note that for uniform circular motion

$$\frac{d^2r}{dt^2} = 0,$$

hence the well-known expression for the central or centripetal acceleration in uniform circular motion:

$$f_r = -r\left(\frac{d\theta}{dt}\right)^2 = -\frac{V_\theta^2}{r}. \tag{1.15}$$

(*b*) Note that the Coriolis acceleration is zero if either V_r or V_θ is zero, i.e., if the velocity is either along or perpendicular to the radius vector.

4. *Equation of conic section in polar co-ordinates: parameter*

In an elementary presentation of orbital motion, the equation of the conic section in the polar co-ordinates *radius vector* r and *true anomaly* v is particularly useful. This equation is easily derived from the property

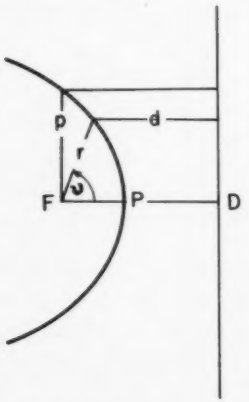

FIG. 1.2—Conic section defined through constant ratio of distances from focus and directrix.

that the conic section is the locus in a plane of all points having a constant ratio e between the distance r from a fixed point F (focus) and the distance d from a fixed line (*directrix*):

$$r = e.d. \tag{1.16}$$

The distance from focus to directrix is

$$DF = d + r\cos v = \frac{r}{e}(1 + e\cos v). \tag{1.17}$$

Here v is counted, in the direction of orbital motion, from the shortest radius vector FP.

The radius vector for $v = \pi/2$, is half the latus rectum or the *parameter* p. Since

$$p = e.DF, \tag{1.18}$$

eliminating DF from (1.17) and (1.18) yields

$$r = \frac{p}{1 + e\cos v}, \tag{1.19}$$

the well known equation of the conic section in polar co-ordinates, referred to the focus; e represents the eccentricity.

Closest approach $FP = r_P = p/(1+e)$ is reached for $v = 0$. Therefore the ratio $(1+e)$ of the parameter p to the distance at closest approach, lies between 1 and 2 for an elliptic orbit. It equals 2 for a parabolic orbit and exceeds 2 for a hyperbolic orbit.

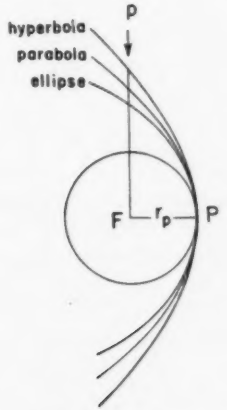

FIG. 1.3—Relation of parameter to closest approach for circular, elliptic, parabolic and hyperbolic orbits.

5. *Elliptic orbit*

The elliptic orbit is of primary importance in astromechanics. Obviously, closest approach and greatest retreat are reached at the end of the major axis.

For $v = 0$ (perihelion, perigee, periastron distance)

$$r_P = \frac{p}{1+e}. \tag{1.20}$$

For $v = \pi$ (aphelion, apogee, apastron distance)

$$r_A = \frac{p}{1-e}. \tag{1.21}$$

The average

$$\frac{r_P + r_A}{2} = \frac{p}{1-e^2}, \tag{1.22}$$

is called the *mean distance*, which equals the *semi-axis major* a hence

$$p = a(1-e^2), \tag{1.23}$$

and also

$$r_P = a(1-e), \quad r_A = a(1+e).$$

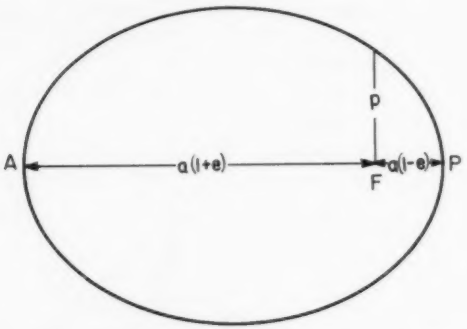

FIG. 1.4—Closest approach, farthest retreat and parameter in elliptic orbit.

Thus for an elliptic orbit, the equation (1.19) in polar co-ordinates, referred to the focus, may also be written as

$$r = \frac{a(1-e^2)}{1+e\cos v}. \tag{1.24}$$

Obviously a represents the *size*, and e the *shape* of the elliptic orbit.

6. *Auxiliary circle; true and eccentric anomalies*

It is often convenient to consider an elliptic orbit as the projection of a coplanar auxiliary circle tangent at the ends of the major axis. For any

location S in the elliptic orbit the projection, or foreshortening, factor k equals the ratio

$$k = \frac{r \sin v}{\sqrt{a^2 - (ae + r \cos v)^2}}, \tag{1.25}$$

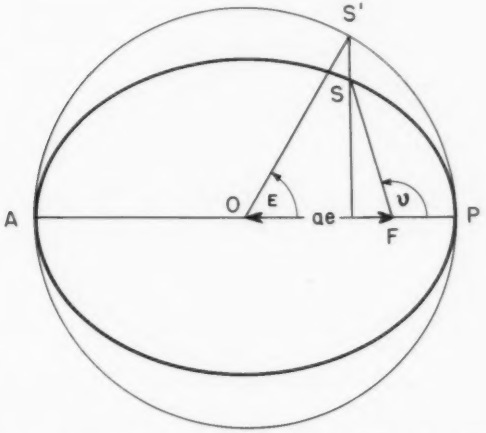

FIG. 1.5—Relation of elliptic orbit to auxiliary circle. True and eccentric anomalies.

which is easily reduced to

$$k = \sqrt{1 - e^2}. \tag{1.26}$$

Corresponding to the *true anomaly* v, the *eccentric anomaly* E is the angle at the centre counted from the major axis to the point S' on the circle from which the corresponding point S on the ellipse is obtained by projection. Referred to the centre, the rectangular co-ordinates of S' are given by

$$\begin{aligned} X &= a \cos E, \\ Y &= a \sin E. \end{aligned} \tag{1.27}$$

The positive x-axis points toward perihelion; the positive y-axis toward $E = \pi/2$. The rectangular co-ordinates of the corresponding point S on the ellipse are given by

$$\begin{aligned} X &= a \cos E, \\ Y &= a \sin E \sqrt{1-e^2}. \end{aligned} \tag{1.28}$$

Equations (1.28) show that the ellipse is symmetrical with respect to the x- and y-axes, respectively. The diameter at $E = \pi/2$ is the minor axis; the length of the *semi-axis minor* is given by

$$b = a\sqrt{1-e^2}. \tag{1.29}$$

Since

$$p = a(1-e^2), \tag{1.23}$$

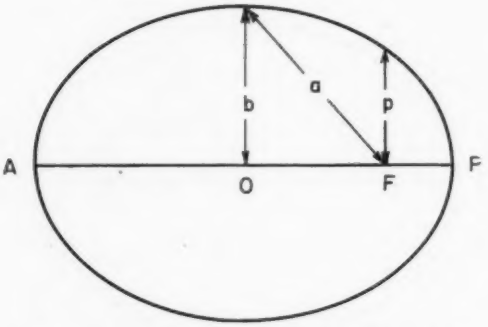

FIG. 1.6—Relation between parameter, semi-axes major and minor in elliptic orbit.

we also find the relation

$$p = \frac{b^2}{a}. \tag{1.30}$$

Obviously, the foreshortening factor k equals the ratio of minor to major axis, or

$$k = \frac{b}{a} = \sqrt{1-e^2}, \tag{1.31}$$

and (1.28) may be written as

$$\begin{aligned} X &= a\cos E, \\ Y &= b\sin E, \end{aligned} \tag{1.32}$$

from which follows the well-known equation, in rectangular co-ordinates, of the ellipse with the centre at the origin:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1. \tag{1.33}$$

7. *Elliptic rectangular co-ordinates*

Referred to the *focus*, the *elliptic rectangular co-ordinates* expressed as

functions of the true anomaly v and of the eccentric anomaly E are related as follows

$$\begin{aligned} r \cos v &= a\,(\cos E - e), \\ r \sin v &= a \sin E \sqrt{1-e^2}. \end{aligned} \tag{1.34}$$

Squaring and adding equations (1.34), we find the useful relation

$$r = a\,(1 - e \cos E), \tag{1.35}$$

from which follows that the radius vector r_b at the end of the minor axis ($E = \pi/2$)

$$r_b = a. \tag{1.36}$$

A convenient relation between v and E is found by eliminating r from (1.35) and the first equation (1.34):

$$\cos v = \frac{\cos E - e}{1 - e \cos E}, \tag{1.37}$$

leading to

$$\frac{1 - \cos v}{1 + \cos v} = \frac{1 + e}{1 - e} \cdot \frac{1 - \cos E}{1 + \cos E}, \tag{1.38}$$

whence

$$\tan \frac{v}{2} = \sqrt{\frac{1 + e}{1 - e}} \cdot \tan \frac{E}{2}. \tag{1.39}$$

8. *Properties involving both foci*

The linear distance of the focus F to the centre is ae. The "other" focus equidistant on the other side of the centre does not appear explicitly in the equation in polar co-ordinates and has no physical significance in astromechanics. It is, however, of significance in the construction of the ellipse which is usually carried out in rectangular co-ordinates. The relation between the distances r and r' from a point on the ellipse to the two foci is of particular interest. We have

$$r = a(1 - e \cos E). \tag{1.35}$$

It is easily seen that for the other "reflected" focus

$$r' = a(1 + e \cos E). \tag{1.40}$$

Hence

$$r+r' = 2a, \tag{1.41}$$

which is the well-known expression showing that the sum of the radii vectores connecting any point on the ellipse with the two foci is equal to the length of the major axis.

Another useful property is the equality of the angles ϕ (angles of "reflection") at a point S on the ellipse between the normal and the radii r and r' to the two foci F and F'. This is proven by reflecting focus F on the tangent. The line joining the other focus F' and the reflected focus F'' is the minimum distance between F' and F'' and therefore

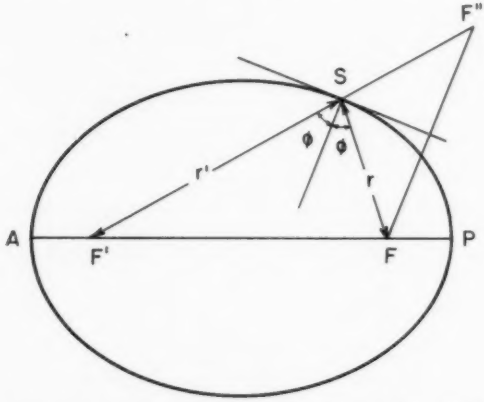

FIG. 1.7—The normal in an elliptic orbit makes equal angles with the radii vectores to the two foci.

passes through the point S. The (mirrored) equality of the two rectangular triangles formed by the focus F, its reflection F'', and the point S, implies the equality of the two angles. Q.E.D.

9. *Parabolic orbit*

For a parabolic orbit $e = 1$, hence

$$r_P = \frac{p}{2}, \tag{1.42}$$

also written as

$$r_P = q. \tag{1.43}$$

Since a parabola is an ellipse with the other focus F' at infinity, the aforementioned equality of angles between the normal and the radii vectores, reduces to the well-known property that rays parallel to the axis of the parabola are reflected through the focus.

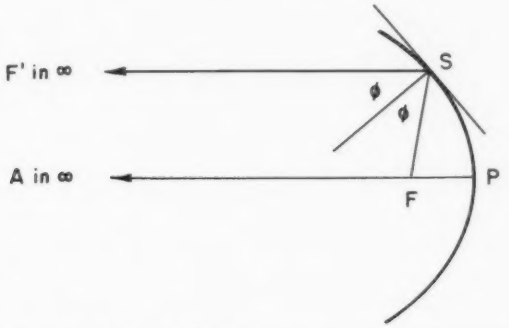

FIG. 1.8—The normal in a parabolic orbit makes equal angles with the radius vector to the focus and the axis of the parabola.

(To be continued)

A MODERN EQUATORIAL SUN-DIAL

By Dean Babcock

Winslow, Arkansas

Introduction. A sun-dial is usually regarded as an instrument for showing the time of day by means of the shadow cast by a gnomon on a plate; and the word is so defined in most dictionaries in common use. Historically, however, a solar device which indicates the time of year is equally important. Indeed, it was probably of greater importance to a primitive people, or in a non-industrial society.

The "sun-dial" of which a geometric construction is described by Vitruvius, in Book IX, Chapter 7 of his classic work on Architecture, was simply the Greco-Roman *analemma*, an instrument of brass or wood, set on a pedestal, showing the meridian, latitude and horizon of the place. Some forms of this instrument could be used to determine the relative length of the day at different times of year; but its primary purpose seems to have been that of indicating the solstices, the equinoctial line, and, more or less accurately, the intermediate times of the year.

Several modern instruments can be used to show the hour angle of the sun, and to determine the true meridian, when the latitude of the place and the declination of the sun are known: notably the Burt Solar Compass, an ingenious though rather complicated instrument, now obsolete but formerly much used by land surveyors in the United States, especially in the survey of public lands.

An equatorial sun-dial designed and made by the present writer and illustrated in this article is believed to embody in a simple way, that is, without moving parts, all or most of the features and functions possible in an instrument which makes use of shadows from the sun. At the time it was made, about 25 years ago, it was designated by the term "heliolabe", a coined word following the analogy of *astrolabe*; and this word will be used occasionally in the following description.

General Description. The heliolabe is a universal equatorial sun-dial, self-orienting and adjustable to any latitude. It is so designed that the shadow cast by the gnomon on the dial indicates the time of day, while the shadow cast by the dial on the gnomon indicates the day of the year.

The instrument consists essentially of three parts:

1. *The Equatorial Ring*, usually called simply the "ring", which, when the instrument is levelled, oriented, and adjusted to the proper latitude, is parallel to the plane of the equator.

2. *The Meridian Plate*, also referred to as the gnomon-, zodiac-, or calendar-plate, or simply "plate", which is in the form of a half circle. It is attached by its outer or curved edge to the inner edge of the ring, in such a way that the straight edge or diameter of the plate coincides with the polar axis of the ring. Consequently, when the instrument is set up, this straight edge or gnomon is parallel to the axis of the earth, and the plate lies in the plane of the meridian.

3. *The Base*. This consists of two disks, pivoted together: a lower disk which may be bolted to a pedestal, and a smaller upper disk bearing a low column to which the edge of the meridian plate is clamped. The upper disk, and consequently the whole instrument, can thus be rotated in azimuth and clamped in the proper position.

Scales and Graduations. The equatorial ring bears an hour circle, reading from 4:00 a.m. to 8:00 p.m. (for high latitudes the entire ring could be graduated, showing the full 24 hours), with the whole hours numbered. The hours are each divided into six equal spaces of 10 minutes each. The fractional parts of the hour are indicated graphically by lines which increase in length in the direction of the augmentation of time (see figure 1 *a*), so that it is impossible or at least unlikely that an observer

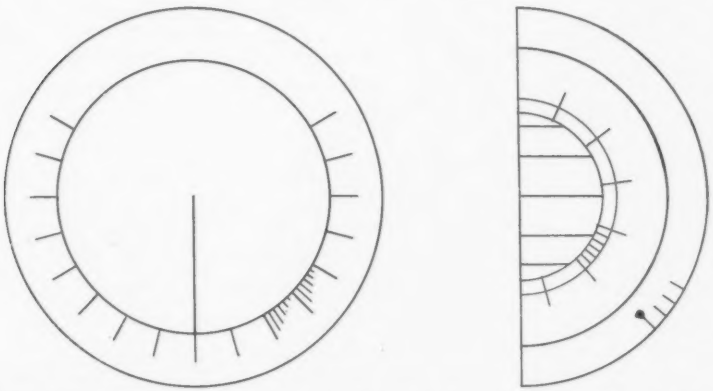

FIG. 1—(a) *left*: The hour circle scale on the equatorial ring; (b) *right*: the longitude graduations.

could make the mistake of reading, for example, 10:10 a.m. for 9:50 a.m. Only the upper surface of the ring is graduated, as the shadow line can be observed on the inner edge of the ring, and conveniently read on its upper side, whether the sun is above or below the equator.

The gnomon plate bears three scales, as follows:

1. Zodiac Scale or Solar Year. The straight edge of the plate receives a shadow from the outer edge of the ring, and this shadow remains practically at the same point on the gnomon edge throughout the daylight hours of a single day. (The rest of the ring's shadow on the plate may be ignored.) This shadow line intersects the axial edge at its centre, or zero point, when the sun is on the celestial equator; and throughout the year its distance from the zero point is measured by the tangent of the sun's declination, multiplied, of course, by the radius of the ring in terms of any small linear unit that is convenient, such as the 60ths of an inch on an engineer's scale. Declination has been converted into solar longitude, so that the actual graduations on the gnomon represent the position of the sun in the ecliptic, according to the sign and degree of the zodiac. Half of the solar year is thus shown on one side of the plate, the other half on the opposite side, the division being at the solstices. Each sign is subdivided into six spaces, each space representing five degrees of solar longitude.

The longitude graduations are carried back on straight lines perpendicular to the edge of the plate, and terminate in a semicircular arc on each side (see figure 1 *b*); or strictly, in two quadrants whose centres are separated by a distance equal to the thickness of the ring. Each of these quadrants represents one of the four seasons, and the four quadrants together constitute a projection of the complete zodiac. The signs of the zodiac are designated by their symbols, and other symbols and markings serve to guide the eye along the observed longitude line to the curved reference line.

2. Calendar Scale or Civil Year. Against the circular arcs just mentioned, and correlated with the longitude scale, is a scale representing the 365 days of the normal civil year. This scale is divided into the months, indicated by their names, and each month into the proper number of days, in groups of five days, to facilitate reading. The day spaces are almost equal throughout this scale, and the method of graduation is similar to that of the hour circle.

3. Latitude Arc. A simple scale of angles, with the zero at the point of intersection of the ring and the plate, is engraved on the limb of the lower quadrant of the meridian plate, or rather, on its semicircular frame. This reads against a pointer or fiducial line on the clamp or column of the base, and permits a direct adjustment to the geographic latitude of the place. In case the latitude is not known it can be determined with sufficient accuracy by means of the instrument itself, any day at noon.

FIG. 2—The equatorial sun-dial.

Corrections for Semi-Diameter of the Sun. The hour angle of the sun is corrected for semi-diameter by calculating the required thickness of the gnomon plate in relation to the radius of the ring.

The zodiac scale, and hence the civil calendar, is corrected for semi-diameter by a slight shifting of the primary declination scale on the polar axis.

(Remark: It has been my experience that any sun-dial which attempts accuracy should be at least approximately corrected for semi-diameter, except those instruments of which the gnomon is a wire or stretched thread).

Limits of Precision. On the present instrument, which is the partial skeleton, so to speak, of a sphere nine inches in diameter, the local solar time can be read, whenever the sun is shining, within two minutes; and often, after a little experience, to the exact minute.

The date can be read to the exact day, except during the periods extending from about 16 or 18 days before each of the solstices to the

same number of days after the solstices, altogether about 65 to 70 days out of the year, when the declination of the sun shows little daily change. However, the reverse procedure, of orienting the instrument by means of a known date, can be accomplished as accurately at the solstices as at any other time.

Observations for orienting the heliolabe should be made before 10:00 a.m. or after 2:00 p.m., or preferably when the hour angle of the sun is at least 40 degrees, plus or minus, as in the determination of the meridian with a surveyor's transit or solar compass.

As the discrepancy between the tropical (solar) year and the calendar year amounts to only one day in every four years, and as the co-ordination of date with solar longitude has been calculated for a mean between leap years, the error of the calendar scale can not be more than half a day; in fact, I was told by a professional astronomer and mathematician who examined the instrument that the theoretical error was considerably less than that, for about eight months of the year.

Conclusion. The heliolabe, like any other sun-dial, shows directly the local solar, or apparent, time. To obtain Standard Time it is of course necessary to apply the equation of time, and to make the correction for terrestrial longitude. Otherwise, no tables and no calculations are necessary in using the instrument; and no accessory instruments are required except a small level for levelling the pedestal.

Incidentally, it may be remarked that in these days of innumerable clocks and radios there are easier and more practical ways of setting a watch to Standard Time than by taking it from the reading of a sun-dial.

This instrument is made of plate brass, the parts fastened together with small brass bolts. It was made almost entirely with a few simple hand tools, the engraving being done "free-hand". It has been taken apart several times for shipping or carrying from place to place; and it has been in actual use, set up out-doors, for at least twelve years. It has proved to be quite durable and remarkably accurate.

A COMPOUND INTERFEROMETER*

By Arthur E. Covington

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Abstract

The formation of a single-lobed, fan-shaped receiving pattern for a radio telescope has been accomplished by combining several individual interference patterns. The antenna described is 600 feet long 185 metres), forms a pattern 2 degrees N-S by 1.2 minutes E-W at an operating wave-length of ten centimetres, and is used for solar noise observations at the National Research Council, Ottawa. Preliminary interpretations of some of the scanning curves of the solar disk suggest that the E-W extent of the radio emissive regions associated with the sun-spots is seldom smaller than 1.5 minutes of arc.

Introduction. A compound interferometer has been constructed at the laboratories of the National Research Council in Ottawa, and is being used to study solar radio emission at a wave-length of ten centimetres. Its mode of operation is derived from the phase-sensitive interferometer first developed by Ryle (1952), and for a given linear aperture it gives the sharpest possible single-lobed fan-shaped antenna pattern (Covington and Harvey). It may be simply described as an interferometer with dissimilar elements: one is a continuous line array which produces a single-lobed pattern; the other is a four-element grating which produces a multi-lobed antenna pattern. The resultant interferometer pattern is given by a multiplication of the two individual antenna patterns, and since the two antennas are placed end-to-end in a straight line so that the phase centres do not overlap, an extra cosine interference term appears as a third multiplying factor. The performance of the antenna system may also be regarded as the selection of one of the fine lobes of the grating antenna by the single lobe produced by the continuous array, accompanied by a further sharpening of the selected grating lobe by the extra interference term.

An aerial view of the interferometer, which is located ten miles south of Ottawa at Goth Hill, is shown in figure 1. The line of antenna elements which comprise the interferometer is oriented due East-West, with an extent from tip to tip of 600 feet (185 metres). The easterly end of this length is occupied by the 150-foot horn which appears in the upper right-hand corner of figure 1. The westerly part has four parabolic cylinders, 8 feet by 10 feet, which form the grating antenna and appear in line

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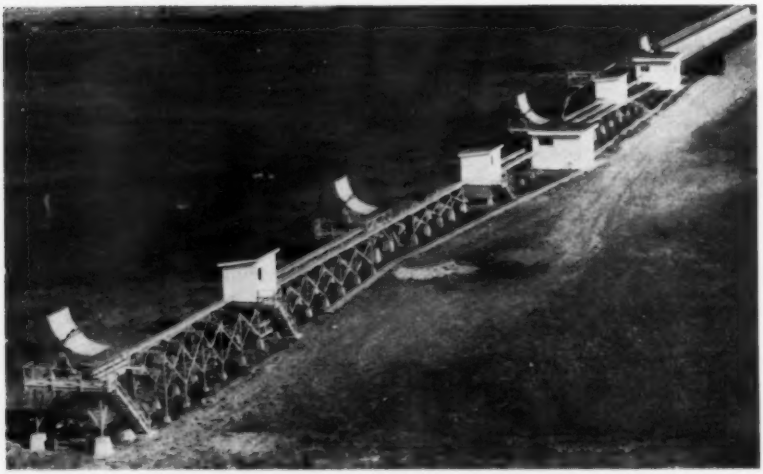

FIG. 1—Aerial view of the 10-cm. compound interferometer, 600 feet long (185 metres), in E-W direction. Upper right-hand corner shows 150-foot slotted waveguide array; lower left-hand corner shows 4-element grating antenna with gaps of 150 feet.

with the array in figure 1. The wooden trestle seen between the grating elements was constructed to support the waveguide transmission lines which provide the electrical radio-frequency connection between the elements. The transmission lines start from each element, are joined together in pairs at junctions which are housed in the small shelters with sloping roofs, and continue to the nearer flat-roofed building which houses the receiver equipment. The copper waveguide transmission lines are of rectangular cross-section, 3 inches by 1.5 inches, and have been placed in insulated wooden boxes in an attempt to keep the feeder system at a uniform temperature under various conditions of weather during the day's observations.

The various antenna elements of the interferometer may be set in elevation about the E-W axis, so that this instrument is used in the manner of a meridian telescope. Each day, near noon, the sun drifts through the receiving pattern of the instrument, which has a single lobe with a half-power width of 1.2 minutes of arc in the E-W direction, and 2 degrees in the N-S direction. Thus, with this fan-shaped antenna pattern, emission from the solar surface is examined in N-S strips. The use of travelling-wave, slotted-waveguide antennas as interferometer elements means that the primary antenna patterns, and hence the resultant pattern, may be placed at various angular positions from the

celestial meridian by a suitable choice of the frequency of waves received. Four narrow-bands of frequencies, in the range 2800 mc./s. to 3000 mc./s., have been placed into service through the use of the upper and lower sidebands of two superheterodyne receivers, and enable four antenna patterns to be formed in the range of 3.5 to 7.5 degrees east of the celestial meridian.

The antenna in its present form has evolved from two earlier stages which have been described elsewhere. Initially the 150-foot slotted waveguide array was constructed and used with a DC radiometer for studies of solar radio emission (Gruenberg 1954, Covington and Broten 1954). Although observations of the solar disk and its features with this instrument proved instructive (Covington and Broten 1954, Dodson 1954), it was very apparent that increased resolution was needed to find the size and position and to examine the structure of the still unresolved radio emissive regions. This need for increased resolution led to the development of the compound interferometer, and the construction of a two-element grating for use with the array (Covington and Broten 1957). Observations again showed that in spite of the four-fold increase in resolution there were still many unresolved radio-emissive regions associated with sun-spots. The resolution was increased by a further factor of two by the use of a four-element grating, observations with which have been carried out since May 1958 (Barber 1958, Covington 1959). Some of the results for this period will be discussed and interpreted, after the formation of the antenna pattern has been discussed.

General Theory. The derivation of the antenna pattern for the compound interferometer, as well as an indication of its mode of operation, will be made by considering the energy picked up by the two dissimilar antenna elements of a simple interferometer, designated A and B in figure 2. The incident wave-front originates from a single distant point-source, and in order to describe the thermal and quasi-thermal sources which are being investigated, it is necessary to assume that the phase of the sinusoidal vibration contains a random variable ϕ_r in addition to the usual time variable ωt, where ω is the angular frequency of the wave, and t the time. Waves of the same frequency, originating from different directions, contain other random phase terms not related to the first. The amplitudes of the voltages induced in the two antennas A and B are functions $A(\theta)$ and $B(\theta)$ of the angle θ, where this angle is between the normal to the line joining the two antennas and the incident ray. This angular displacement θ of the ray introduces equal but opposite phase changes in the two waves picked up by the two antennas. This

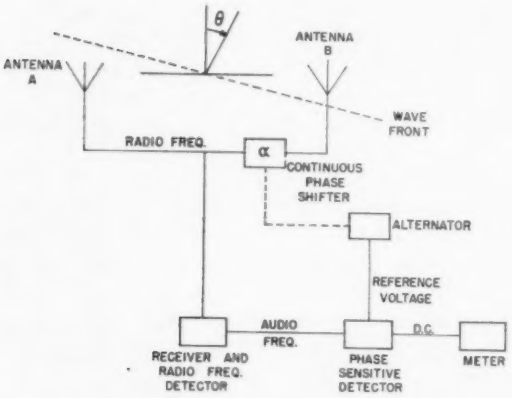

FIG. 2—Components of phase-sensitive interferometer.

phase change equals $\pi\ (D/2\lambda)\theta$, where D is the distance between the two antennas and λ is the wave-length. The two voltages travel through two transmission lines of equal length to a junction, are added together, with a loss designated by k, and then travel to the receiver. In one of the interferometer arms there is a continuously rotating phase shifter which changes the phase of one voltage with respect to the other at a very slow rate, designated by the frequency a. Thus the radio frequency voltage delivered to the receiver, expressed in complex vector notation involving the use of the previously defined variables is

$$V(\theta) = k\left[A(\theta)\exp\left(-\pi\frac{D}{2\lambda}\theta\right) + B(\theta)\exp\left(\pi\frac{D}{2\lambda}\theta + \alpha t\right)\right]\times \quad (1)$$
$$\times \exp(j\omega t + \phi_r)$$

The power $P(\theta)$ delivered to the receiver is found from the multiplication of $V(\theta)$ by one-half of its complex conjugate, and consists of three terms

$$P(\theta) = \frac{k^2}{2}\left[A^2(\theta) + B^2(\theta) + 2A(\theta)B(\theta)\cos\left(\pi\frac{D}{\lambda}\theta + \alpha t\right)\right] \quad (2)$$

The first two terms consist of squared quantities independent of the low frequency a, while the last term consists of the product of three factors, and is a function of a. It is this last term, with a finite value of D, which gives rise to the compound interferometer pattern. After the received power has been amplified by the superheterodyne receiver to a suitable

level and the radio frequency variations which serve as a carrier for the received power have been removed, the wanted fluctuating power represented by the last term in Eq. (2) can be separated from the steady power represented by the first two terms. This is accomplished by passing the rectified receiver output through an amplifier designed to accept only signals with the modulation frequency generated by the operation of the rotary phase shifter. After this, it is necessary to remove "at" from the cosine factor so that the only part remaining is one relating to the angular variable θ. This is done by passing the selected low-frequency signal through a phase-sensitive detector driven in synchronism with the rotary shifter (see figure 2). The mathematically equivalent operation performed on the fluctuating power term in Eq. (2) is a multiplication of that term by cos at, and subsequent expansion as a sum of two new terms. The arbitrary phase angles which arise in actual practice have been assumed to be zero, so that the compound interferometer pattern as given by the appropriate expansion term is

$$k^2 A(\theta)\, B(\theta) \cos\left(\pi \frac{D}{\lambda} \theta\right) \tag{3}$$

A schematic of the asymmetrical interferometer elements used in the compound interferometer is shown in figure 3. The rectangle A represents

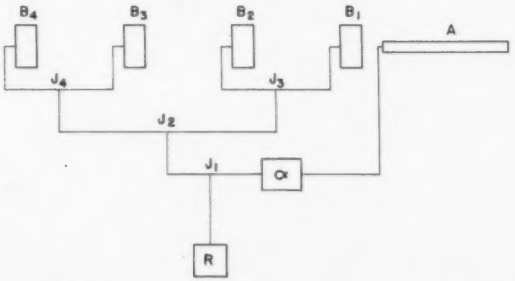

FIG. 3—Schematic of the antenna elements comprising the compound interferometer: A—continuous array; $B_1B_2B_3B_4$—grating elements; α—continuously rotating phase shifter; R—receiver; $J_1J_2J_3J_4$—junctions in waveguide transmission line.

the aperture of the long array, while the grating elements—short sections of a parabolic cylinder—are represented by B_1, B_2, B_3, and B_4, and are placed with gap separation equal to the length of the array. Waveguide transmission lines, with the rotary phase shifter placed in the transmission line coming from the array, bring the collected energy to the receiver via

a series of junctions J_1, J_2, J_3, and J_4. The junctions contain removable waveguide tees or elbows so that it is possible, with a DC radiometer, to receive energy from the array, A; or, with a phase-sensitive radiometer, to use the following combinations of antennas: A with B_1, A with B_1B_2, and A with $B_1B_2B_3B_4$. The successive use of these four combinations of antennas to scan the sun within an interval of one-half hour is possible because of the beam-shifting feature of the slotted waveguide arrays, and results in the formation of the different antenna patterns at various angles from the meridian. These angles of maximum reception east of the meridian are usually defined as "squint angles".

The mathematical expressions for the resultant antenna patterns derived from the successive use of the useful combinations of elements in the grating, as outlined above, will be developed from the basic formula for the phase-sensitive interferometer given by Eq. (3). For the present, it will be assumed that the waveguide feeders are of equal length; that the various arbitrary phases in the extra interference term have been reduced to zero, and that the correct radio frequency defining the squint angle has been chosen. For convenience, in this ideal case the following substitutions are made.

$$x = \pi \frac{D}{\lambda} \theta,$$

$$A(\theta) = \frac{\sin x}{x},$$

$$B(\theta) = 1.$$

The interference term involving the B_1 element is $\cos x$,
that involving the B_2 element is $\cos 3x$,
that involving the B_3 element is $\cos 5x$,
and that involving the B_4 element is $\cos 7x$.

The resultant compound interferometer pattern is derived from the appropriate sum of the single patterns which arise from the operation of each of the grating elements as a phase-sensitive interferometer. As before, in passing through a tee junction, a voltage loss of k is involved. Thus, the power pattern $P(x)$ arising from the array operating with B_1 is given by the equation

$$P(x) = k^2 \frac{\sin x}{x} \cos x \tag{4a}$$

$$= k^2 \frac{\sin 2x}{2x};$$

from the array operating with B_1 and B_2 is given by the equation

$$P(x) = 1k^3 \frac{\sin x}{x} (\cos x + \cos 3x) \tag{4b}$$

$$= 2k^3 \frac{\sin x}{x} \cdot \cos x \cdot \cos 2x \tag{4c}$$

$$= 2k^3 \frac{\sin 4x}{4x} ;$$

from the array operating with $B_1B_2B_3$ and B_4, and is given by the equation

$$P(x) = 1k^4 \frac{\sin x}{x} (\cos x + \cos 3x + \cos 5x + \cos 7x) \tag{4d}$$

$$= 4k^4 \frac{\sin x}{x} \cdot \cos x \cdot \cos 2x \cdot \cos 4x \tag{4e}$$

$$= 4k^4 \frac{\sin 8x}{8x} .$$

The cosine terms in Eqs. (4a), (4b) and (4d) represent basic interference patterns which may be combined mathematically to give new cosine multiplying factors in an alternative mathematical description of the resultant antenna pattern; e.g. Eq. (4d) becomes Eq. (4e). This latter description suggests the name "compound interferometer" for this antenna, with its particular configuration of elements.

It is also apparent that a doubling of the aperture of the grating by the use of twice as many grating elements will reduce the principal lobe of the antenna pattern by one-half. This is represented mathematically by the introduction to the original antenna pattern of a single cosine multiplying factor, with amplitude $2k$; for example, Eq. (4c) becomes (4e) with the multiplier $2k \cos 4x$. The phase term of the new multiplier is twice that of the highest phase term in the product expression for the original antenna pattern. The compound interferometer patterns produced by gratings of various numbers of elements have the same functional form, namely $\sin x/x$, but have different widths of the main lobe, corresponding to the linear extent occupied by the whole antenna.

In the construction of the actual compound interferometer it was found convenient to introduce a small spacing between the end of the grating and the array. This has the effect of modifying the sidelobes so that they are predominantly negative, and in scanning the solar disk these give

rise to an apparent negative value for the temperature of the sky adjacent to the sun. The theoretical pattern calculated from measured dimensions of the antenna is shown in figure 4.

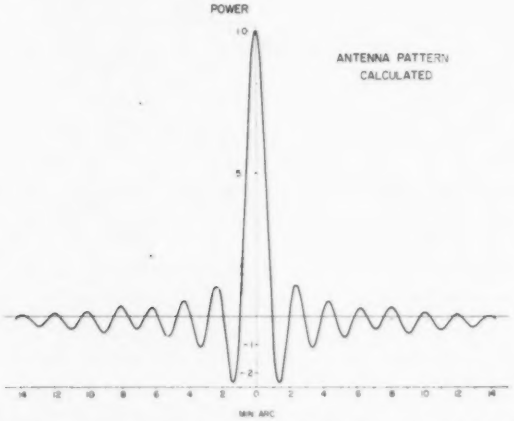

FIG. 4—Theoretical antenna pattern for the 600-foot compound interferometer.

Adjustment of the Antenna. It is impossible to observe the receiving pattern of this or any large antenna in the usual manner by recording the transmissions from a distant movable transmitter, so that complete reliance must be placed upon the quality of the components and the accuracy of adjustment. The observation of drift curves of natural point-like sources which approximate very closely the calculated antenna pattern is very fortunate, and gives confidence in the performance of the antenna. In the formation of this compound interferometer pattern, it is necessary that the maxima of the three separate elemental interferometer patterns (see Eq. (3)) be formed at the same squint angle, in order to obtain a single-lobed pattern. The position of the array pattern is controlled by the frequency of the operation, and, unfortunately, by its temperature; the grating pattern is controlled by the spacing of the elementary antennas and by the phase length of the feeders; finally, the angular position of the interference term is controlled by the phase length of the array feeder, as well as by the mechanical setting of the alternator which is driven from the rotary phase shifter and in turn drives the zero beat detector. Some of the physical factors which enter into the formation and control of these three patterns will be discussed.

The long array pattern which forms the foundation for all subsequent

patterns has been shown to be very satisfactory. The grating pattern is formed by small elements so that they could be tested individually in the usual way, and later placed into the positions of the grating configuration. All these five antenna elements are connected together, by means of waveguide transmission lines and tee junctions, so that the distances from the phase centres of each of the antennas to the receiver are all equal. Referring to figure 3, it can be seen that certain sections of the lines are used in common by two grating elements, and that there is a small length of line from junction J_2 to junction J_1 to ensure that the lengths of the lines from the receiver to all phase centres are approximately equal. The phase centre of the long array is taken to be at the midpoint of the array, so that the section of waveguide from this point to the terminal of the array serves the dual purpose of slotted waveguide antenna and feeder.

The physical measurement of the feeder lengths was made without striving for an accuracy greater than that which could be realized readily. This was done because the equivalent path length includes variations related to the varying cross sections of the waveguide, as well as variations in the overall length. Both can be allowed for in the final electrical adjustments by means of phase shifters in the waveguide feeders. At first the four feeders of the grating were adjusted to integral values, but not necessarily equal integers, by using phase corrections which had been determined experimentally. These corrections were found with reference to one feeder, which is regarded as a standard, and were derived by measuring the relative standing wave pattern produced by sending a strong signal down each line, and by reflecting it back from a short which was substituted for the grating elements. The testing of a single line to each of the grating elements is made possible by replacing one or more of the tee junctions with an appropriate elbow or elbows.

There is, of course, a difference in the electrical lengths of the various arms, equal to some exact number of half-wave-lengths, which cannot be detected by means of standing waves. This gives rise to the formation of several different and unwanted antenna patterns. These half-wave-length differences can be represented analytically by the polarity ($+$ or $-$) of the elementary interference patterns generated by the array when operating with given grating elements (definitions for eqs. (4)) and the several antenna patterns calculated. Observations of actual point-like sources are required to select the single desirable pattern from the other patterns. When advantage is taken of the previous correct operation of the array operating with two grating elements, then only four of the eight combinations of half-wave-length differences need be tested in

this manner. The correct pattern is given by the voltages represented by the combination of signs such as $+ + + +$ which appear in Eq. (4d), where the first sign refers to the polarity of cos x, the second sign to the polarity of cos $3x$, etc. The three undesired antenna patterns may be represented by sums of cosine terms designated by the combinations of signs: $+ + + -$, $+ + - +$, and $+ + - -$. The compound interferometer patterns arising from the correct and the three incorrect combinations of single patterns are shown in figure 5, the designated order com-

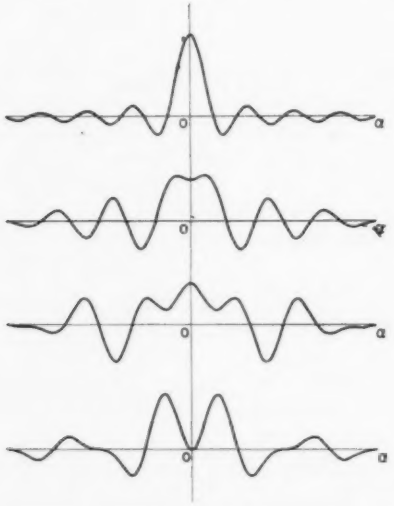

FIG. 5—Theoretical antenna patterns arising from various combinations of exact half-wave-length differences in feeder lengths to antenna elements B_3 and B_4.

mencing at the top and proceeding downwards according to the outlined scheme. As previously mentioned, the identification of the particular physical configuration of feeder lengths with the correct pattern represented by all positive signs has been possible only through the appearance of a point-like sun-spot on the solar disk. During its existence on the disk, the most undesirable pattern with the two lobes was recognized and provided the key for complete identification of the series of patterns. The pattern arising from the combination $+ + + -$ has been used, and found to produce drift-curves which are very hard to distinguish from the correct one.

The adjustments required to eliminate the arbitrary phase constants of the cosine interference terms assumed in the theory to be zero, have proven to be the most critical, and contain two factors. These are: the mechanical setting of the alternator (relative to the slowly rotating radio-frequency phase shifter) which generates the driving voltage for the zero-beat detector and the adjustment of a piece of dielectric in the waveguide line connecting the array to the receiver junction. Both of these settings give rise to fixed phases which are added together and appear in all the phase terms of the cosine interference patterns: in Eqs. (4a), (4b) and (4d). Analysis shows that when the pattern is presented in the product form, the adjustable, fixed phases appear only in the phase of the cosine term which contains the overall length. Small values of this phase term introduce an asymmetrical shift of the sidelobes of the correctly phased antenna pattern, raising those on one side while lowering those on the other side (Covington and Broten 1957). This error in the drift-curve of the solar disk introduces a corresponding raising and lowering of the value of the apparent sky emission adjacent to the solar disk, and values as small as five degrees can be detected and corrected.

The effect of differences in temperature between the various elements is important, and efforts have been made to keep important parts of the structure at the same temperature. The waveguide feeders have been placed in insulated boxes, with flexible waveguide sections placed at appropriate places to relieve stresses due to thermal expansion, as well as stresses which occurred during installation. A polyfoam cover has been placed in the opening of the horn which collects the energy for the long array, and this cover provides thermal stability for short periods of time during rapid temperature changes introduced by variable cloud cover. From summer to winter the ambient temperature ranges from $+30°$C. to $-20°$C., and the accompanying changes in the waveguide dimensions produce a shift of the squint-angle of the slotted waveguide array of about five minutes of arc. This shift is offset by introducing a frequency shift in the local oscillator, which in turn shifts the band of frequencies received, and hence the squint-angle of the antenna pattern. The exact value of the frequency shift is derived from calculations using the measured temperature at the centre of the waveguide array (Gruenberg 1954).

Seasonal changes in the dimensions of each of the feeder arms also alter the path length travelled by each of the signals picked up by the antenna elements. These changes, except for small differential changes appearing from time to time, are cancelled out in the elementary inter-

ference patterns. No attempt has been made to compensate for the residual variations related to temperature differences between grating feeders since other phase variations between grating and array were found to be more critically related to the ambient temperature and other unknown effects. The best evaluation, and hence control of this latter parameter, was finally obtained from actual measurement of the phase difference between the length of the feeder going to the array and the length of the feeder going to grating element B_1. This involved connecting the two lines together at the two antennas, A and B_1, in order to form a long loop of waveguide and to insert a signal from a generator at the midpoint of the loop. Thus, by measuring the balance between the two halves of the loop, it is possible to measure the day-to-day variations in the phase lengths of these two feeders, and hence derive a daily correction. A further temperature correction is indicated because part of the feeder which consists of one-half of the slotted array itself, is usually at a temperature elevated in comparison with that of the array feeder. This temperature difference is measured each day and a correction applied.

The principal requirement of the superheterodyne receiver in its relationship to the antenna pattern is the maintenance of the frequency of the upper and lower side-bands. This means that the frequencies of the local oscillator and of the i-f amplifier should be stable. The necessary stability of 1 part in 10^5 for the local oscillator has been achieved by coupling the low-Q cavity of the reflex klystron into a high-Q cavity, and by placing both in an oven with accurately maintained temperature. The existing i-f amplifier is of conventional design, and the possibility exists that its frequency may have drifted slightly over a long period of time. Improvements to this are possible and necessary for long-term operation. Since the antenna patterns shift in angle as a function of frequency, the bandwidth of the receiver should be as narrow as possible to have minimum blurring effect on the pattern of the antenna, and yet wide enough to pass sufficient power. A 0.5 mc./s. bandwidth has been used, which produces a 10 per cent. broadening of the calculated beamwidth. It has also been found necessary to determine accurately the phase-shift through the 15-cycle amplifier.

(To be concluded)

CANADIAN SCIENTISTS REPORT—XX

CONTRIBUTIONS OF DOUBLE STAR ASTRONOMY

By R. M. Petrie

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The study of double stars is an important part of modern astronomy, contributing essential information to our knowledge of the physical properties of stars and of stellar evolution. It is appropriate that a discussion of double stars should be included in the Canadian Scientists Report Series because Canadian astronomers have long been important contributors to this branch of astronomy. Beginning with the significant studies at the Dominion Observatory between 1907 and 1911 the discovery and investigation of spectroscopic binaries have been actively continued at the Dominion Astrophysical Observatory, and later, the David Dunlap Observatory. The most recent catalogue of such binaries lists 480 systems for which orbital details are known; of these 175 or thirty-six per cent. are credited to Canadian astronomers.

Double stars constitute an important part of the stellar population. Among stars in general one in every eighteen is seen as a double (or multiple) with the 36-inch refractor of the Lick Observatory, according to Aitken. We must however remember that a great many double stars are too far away, or the components are too close together, to be separated into visual doubles, even with powerful telescopes. Systems of small dimensions are found to be double from spectroscopic observations; Campbell, in considering such systems, estimated that at least one star in four is attended by a visible, or an invisible, companion. More recent studies tend to increase that estimate. A statistical survey of radial velocities just completed at Victoria gives as preliminary values, the following percentages of spectroscopic binaries among stars in general:

B stars	49 per cent.
A stars	55 per cent.
G stars	50 per cent.
K stars	50 per cent.
M stars	59 per cent.
All types	55 per cent.

From this table we reach the surprising conclusion that about one-half of the stars, so far studied with the spectroscope, are double!

Statistics of stars relatively very near the earth agree with the above results. Van de Kamp has listed 42 stars known to be within 16 light-years of the sun. Of these 15 are double or multiple, nearly 40 per cent.

The high frequency of double stars underlines their importance. Theories of star formation and stellar evolution ignore them at their peril. Binary stars are as common in the galaxy as single stars and must be regarded as a natural mode of stellar existence and development.

The first star to be recognized as double was Mizar, the famous object in the "handle" of the "Big Dipper". The discovery was made by Riccioli, about 1650. By a remarkable coincidence the system of Mizar also gave us our first spectroscopic double. The brighter component of Riccioli's visual pair was found by Pickering, in 1889, to be a spectroscopic binary.

Double stars were first considered by astronomers as interesting, but not very important, curiosities. Serious study dates from 1779 when Mayer published a catalogue of 80 double stars and discussed their nature, asserting that the fainter star revolved about its brighter companion. Also, in 1779, Sir Wm. Herschel began systematic observations on selected double stars for the purpose of measuring the minute parallactic displacements caused by the earth's annual motion around the sun. He hoped thus to be able to measure stellar distances. Herschel failed in his original aim but, in failing, made a discovery of first importance. After 25 years of patient measurement he discovered (1803) that Castor, and five other double stars, showed orbital motion consistent with the law of gravitation. Thus the scientific investigation of double stars began. Today we catalogue some 20,000 binaries seen to be double in powerful telescopes. For the great majority of these the period of revolution is so long that we know very little about the orbital motion; details are known with satisfactory precision for only some 120 visual binaries.

It is obvious that double stars must exist in profusion beyond our power to see them as two separate stars. Such pairs generally would have orbital radii of a few million miles and have periods of revolution of a few days, or perhaps weeks. The orbital velocities would be great enough to be detected with a spectroscope and this in fact led to the discovery in 1889, referred to above. The brighter component of Mizar was shown to be composed of two, nearly equal, stars describing orbits about their barycentre in a three-week period. Discovery in this field has been rapid; we know of the existence of at least 2000 spectroscopic binaries, and orbital dimensions have been determined for nearly 500 systems.

In addition to discovery by means of the spectroscope, double stars sometimes reveal themselves through a periodic fluctuation in brightness. These are the eclipsing binaries and for this special group the orbital planes happen to be oriented in the direction of the solar system and the stars therefore eclipse one another (as seen from the earth) during the course of their orbital revolutions. Large numbers of eclipsing binaries

have been discovered with the aid of the very efficient modern photometric techniques. A catalogue, published in 1958, lists nearly 2800 known eclipsing double stars. Eclipsing binaries may usually be investigated also with the spectroscope since the periods of revolution are ordinarily a few days and the velocity variations are correspondingly large.

Observations of Double Stars

Visual Binaries. The observations are straightforward and relatively simple; they are usually made visually at the telescope with a micrometer or, alternatively, upon direct photographs. One measures the distance (in angular units) between the component stars, and the direction of the line joining them. A separation of the stars of about one-tenth of a second can be measured visually in favourable conditions; if the pair has a smaller separation it is possible to measure them with the help of

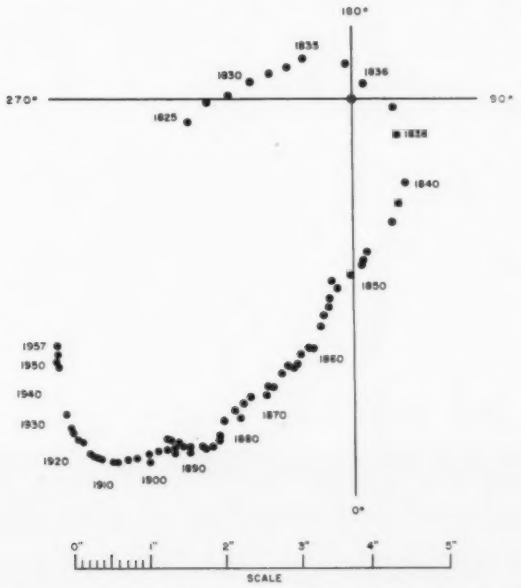


FIG. 1—The visual binary γ Virginis. The filled circles show the position of the fainter star of the pair relative to the brighter, indicated by the open circle. Note the rapidity of the motion between 1830 and 1840 when the components are at their closest approach to one another and the leisurely movement between 1900 and 1920 when they are at their greatest separation. An ellipse drawn through the filled circles gives the relative apparent orbit and leads to a determination of the elements.

a double-star interferometer. Photographs normally are used only of pairs with separations greater than one second of arc.

When the observations have been continued for a sufficient length of time one may draw the apparent path of one star about the other and find the apparent orbit which is elliptical in shape. The apparent ellipse gives us the so-called elements which are the fundamental information pertaining to the motion of two stars about their barycentre in accordance with the law of gravitation. The elements are: the period of revolution in years, and the time when the stars are at their closest approach to one another; two numbers specifying the shape and size of the orbit; and three numbers describing the orientation of the orbit with reference to the orbit of the earth about the sun. The orbital elements allow us to predict the appearance of the double star at any time and to derive some information about the masses of two stars.

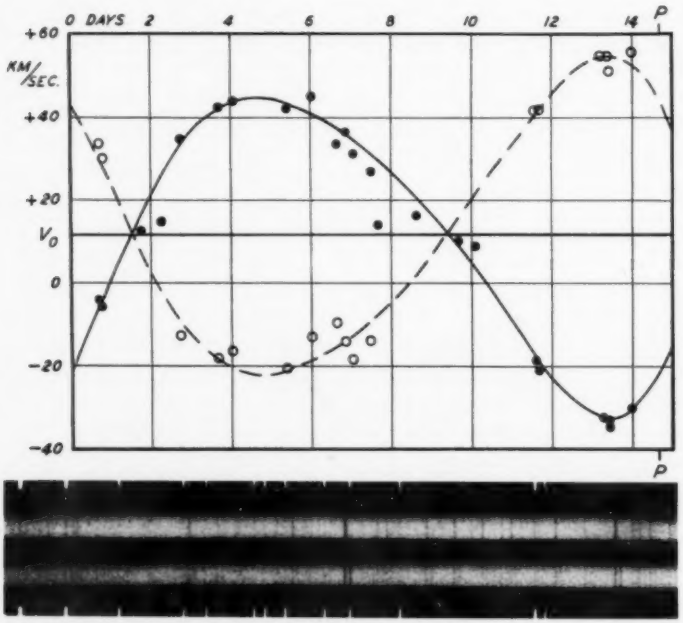


FIG. 2—Spectra and line-of-sight velocities of H.D. 171978. The spectra show lines of both components of this spectroscopic binary when the velocities are at their greatest difference (at elongations) and single superposed lines when the velocities are the same (at conjunctions). The curves drawn through the observed velocities of the brighter component, filled circles, and the fainter component, open circles, give the spectroscopic elements.

Measurements of the absolute, rather than the relative, motions of the component stars are most valuable and should be made whenever practicable. The additional information gives the mass-ratio and individual masses of each star; without it we can compute only the total mass of the system. Also the brightness ratio of the members of a binary system should be measured in order to give information on the role of double stars in the stellar sequence and on the relation between stellar mass and stellar luminosity.

Spectroscopic Binaries. The basic observations are line-of-sight velocities determined with the aid of a spectrograph. When the orbital period is determined by trial, the assembled velocities give a smooth curve whose shape and range determine the "elements". The spectroscopic elements are: the period of revolution, and the time when the stars are closest to one another; the line-of-sight velocity of the barycentre of the system, and the range in the velocity; and two numbers specifying the shape of the ellipse and its orientation in the orbit plane.

Special interest attaches to those spectroscopic doubles for which the spectra of both components can be photographed. We may then find the mass-ratio and the individual masses (reduced by a projection factor) and also the brightness ratio. These data are of prime value in studies of stellar structure and stellar evolution.

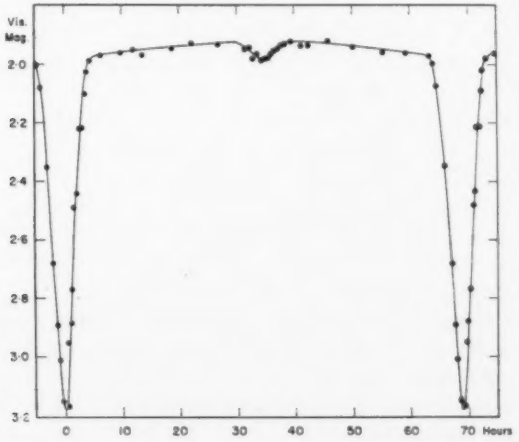


FIG. 3—The light curves of Algol. The large drop in light, nearly seventy per cent., occurs when the brighter star of the pair is eclipsed. A smaller drop is observed one-half a period later when the fainter star is in eclipse. The slow variation between eclipses is explained by reflection and non-sphericity of the stars. The curve drawn through the observed light changes (filled circles) leads to the eclipse elements.

Eclipsing Binaries. Observations are made with a stellar photometer, giving the brightness of the binary. When assembled on the light (orbital) period the observations give a curve showing the decrease, and recovery, of the light during the eclipses. Light curves often show, also, variations caused by reflection and by the distortions of the stars produced by tidal, and rotational, forces. The principal light elements are: the period of revolution and the time of the middle of an eclipse; the radii of the stars in units of the orbital radii; numbers specifying the shape of the orbit, its orientation and the inclination of the orbital plane; and the brightness ratio of the stars forming the eclipsing binary.

Absolute Dimensions

Double-star data are of general interest in themselves but of more value to astronomy are certain absolute quantities, i.e. the masses and radii of the member stars and their true separation from one another, in linear units. Each type of double star lacks some determinate observation which prevents a complete solution without the provision of some additional information. For example, we can measure the separation of the components of a visual binary only in angular units and we do not know their true separation unless the distance to the binary is known. Again, we cannot tell the true velocity range of a spectroscopic binary unless we know the inclination of its orbit relative to the line-of-sight, since we can observe only the projected range of velocity. The eclipsing binaries give the relative radii of the component stars but their actual radii cannot be found unless we know also the true separations.

It is possible, sometimes, to combine two modes of observation so as to remove some of the indeterminacy. If, for example, we can obtain radial-velocity curves of the stars in a visual binary we can fix the true scale of the system and find the mass of each star. The system of Capella has been fully determined by such a combination of observations, but is a rare case, because the velocity variations are generally too small to measure with the required accuracy.

The most frequent, and valuable, opportunities are presented in the spectroscopic observation of eclipsing binaries, if the spectra of both stars can be photographed. The combination of photometric, and spectroscopic, elements then enables us to calculate the masses, radii, luminosities, and mean densities in absolute units. This is our only source of such fundamental information (except for the sun) obtained directly from observed quantities; it is therefore of the greatest value to astrophysics and cosmogony.

Some Applications

We may now consider some general applications to astronomical progress, assuming that the necessary observations of double stars have been made and the calculation of orbital elements carried out. It is necessary, in this brief account, to ignore the many complexities in double-star systems which are both the trial and delight of the double-star astronomer and to assume that all has gone well and the fundamental data are available.

The Mass-Luminosity Relation. The relation between the mass and the radiative power of a star appears to be a fundamental property of stellar bodies. The relation is of profound interest because it helps us to understand the properties of stellar interiors, the sources of stellar energy, and the probable ages and course of evolution of stars. The observational material which led to the discovery of the relation, and to which the theoretical explanations must conform, comes entirely from visual binaries with known orbits and distances, and from eclipsing binaries observed also with the spectrograph.

The number of binaries supplying first class dimensions has never been great and some of the data are not entirely accurate. A few years ago it became possible to derive a mass-luminosity relation entirely from spectroscopic binaries not known to be eclipsing and quite independent of the material used previously. The method involved the measurement both of the light-ratio, and of the mass-ratio, of the component stars and was relatively free from the pernicious effects of errors in distance determinations which weaken much of the data supplied by visual binaries. A new mass-luminosity relation was derived at Victoria from nearly 100 two-spectrum binary systems and was verified by the most reliable data from eclipsing systems. This application added to the observational information on the relation between mass and luminosity especially in the region of large stellar masses.

Apsidal Motion. The stable and predictable orbit of a double star presupposes the ideal conditions of the two-body problem of celestial mechanics, i.e. spherical stars with symmetrical internal density distributions. In some spectroscopic binaries the stars are distorted by tidal and rotational forces and orbital perturbations result. The principal variation is a slow turning of the major axis of the orbit in its own plane; the effect is called apsidal motion and can be detected by observations of the line-of-sight velocity curve. A good example is furnished by the binary H.R. 8800, a pair of high-temperature stars revolving about their mutual

centre of gravity in 3¾ days. The system was observed at Victoria in 1918 and again in 1958. The major axis of the orbit had changed by about 90° during the 40-year interval and an apsidal period of 154 years was established.

The observational evidence on apsidal motion in binary stars, although not yet plentiful, holds future promise. The distortions of rapidly rotating, and relatively close, pairs of stars depends, for a given configuration, upon their internal density distributions. The observation of apsidal motion thus tells us something about the internal architecture of stars and is the most direct evidence we can obtain on this subject at the present time.

Luminosity and Velocity Standards. Direct measures of distance are made successfully only to relatively nearby stars. At distances above 500 light-years the direct measurements are impotent and yet, for a proper exploration of the galaxy, we must know the distances of stars at several thousand light-years. One must use indirect methods which essentially depend upon a knowledge of the true brightness of stars. It is difficult to obtain the necessary standardizing material for the high-temperature stars which must be observed for galactic studies because, generally speaking, none is within reach of the direct trigonometrical method of measuring distance.

Visual binaries are very useful in supplying luminosity standards for the hotter stars. We search for binaries in which a hot star is paired with a cooler star (rather like the sun) for which methods of estimating luminosity are known. Having determined the luminosity of the cooler star we may at once find that of its hotter attendant since both are at the same distance from us. The eclipsing binaries also are helpful in this problem. A number of systems are known composed of high-temperature stars and fully observable with photometer and spectrograph, so that radii and luminosities may be calculated from observational data.

Calibration of the luminosities of the hotter stars is absolutely necessary before we can interpret our observations of stellar motions and distribution in terms of galactic structure and galactic dynamics. We could not make the calibration with acceptable precision without the data from double star astronomy.

A similar situation confronts us when we attempt to verify the accuracy of our velocity determinations on the hotter stars, again required in galactic studies. The velocity of a solar-type member of a visual binary may be measured with certainty, because we can verify our methods from the known velocities of bodies in the solar system. If the cooler star in the visual binary has a high-temperature companion our measures of

its velocity can then be checked since both component stars will have essentially the same velocity. In this way we can confirm the methods of velocity measurement for the high-temperature stars.

The importance of a homogeneous velocity system for hot and cool stars alike can scarcely be overemphasized. The celebrated K-term in the velocities of B stars, the relativity "red-shift" in the velocities of massive hot stars, the motions of the interstellar gases, and the differential motions in star clusters and associations are of importance in studies of the galactic structure and of the formation of stars. Every one of the above-mentioned effects depends upon the assurance that the measured Doppler shifts do, in fact, refer to line-of-sight velocities and are not merely errors introduced in our observing or measuring procedures. Thus binary stars are of importance in fields far removed from double star astronomy in itself.

The visual binary A.D.S. 2559 is a good example of a system useful in the above applications. It comprises a high-temperature star (B3) of apparent magnitude 7.4, and a solar-type star (G5) of apparent magnitude 8.5. The separation is 3.5″ so that the stars may be observed separately with a stellar spectrograph. The absolute magnitude of the solar-type member is found to be —0.9, using known luminosity criteria, while its velocity is easily measured as 14.9 km./sec. of approach. Thus we know that the absolute magnitude of the hotter component is —2.0 (more than 500 times the brightness of the sun) while its line-of-sight velocity must be close to 15 km./sec., of approach.

The ζ-Aurigae Stars. The star ζ Aurigae, easily visible to the unaided eye, was found to be a spectroscopic binary more than 50 years ago. The period of revolution is nearly three years (972 days), very long for a spectroscopic binary and indicating a very large orbit. Astronomers were astonished therefore, in 1930, to find that the star was also an eclipsing binary and were further intrigued when it became known that the eclipse is total for 38 days! It was at once recognized that one component of the system must be a star of extraordinarily large dimensions.

THE ζ-AURIGAE STARS

	Period	Total Eclipse	Partial Eclipse	Atmospheric Effects	Radii in solar units		Next Eclipse
					Cool star	Hot star	
	days	days	days	days			
ζ Aurigae	972	38	1.5	10–40	200	5	April 1961
32 Cygni	1148	13	4	40	200	3	April 1962
31 Cygni	3802	61	2	70	150	5	1962
VV Cephei	7400	450	22	365+	1200	10	1977

Several systems of this general nature are now known, and have provided new avenues to the study of supergiant stars and the nature of their outer layers. The systems now recognized are ζ Aurigae, 31 Cygni, 32 Cygni, and VV Cephei. All are quite bright stars as seen from the earth and all lie within 3000 light-years of the earth so we suspect that systems of this sort are not uncommon among the stellar population. The main characteristics of these remarkable binaries are tabulated on page 37.

The four systems are different and yet have remarkable similarities. In each a relatively cool supergiant of enormous size is accompanied by a hot star of normal dimensions. The supergiant is so large that the hot star is merely a point by comparison and when eclipsed, remains behind its gigantic companion for weeks or months.

Observations of the ζ-Aurigae stars tell us much about the sizes, masses, densities, and luminosities of cool supergiants; information of this sort

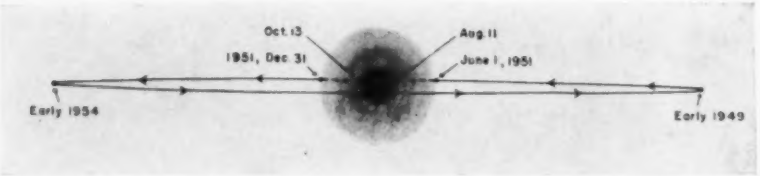


FIG. 4—The system of 31 Cygni. The motion of the small hot star, black dot, about its gigantic companion is shown. The shaded area around the cool giant star represents its outer atmosphere behind which the hot star can be observed before and after total eclipse.

cannot be obtained from single stars. In addition we learn about the outer layers of supergiant stars by observing the light of the hot star as it passes through successive layers of the atmosphere of its companion just before, and after eclipse. The outer layers are found to be very tenuous and to be very extensive. They are in violent and turbulent motion far stormier than the worst terrestrial tempest. In ζ Aurigae, for example, we found a calcium "wind" of some 25,000 mi./hr. blowing thirty million miles above the surface of the star; this rose to a gale of 54,000 mi./hr. at a height of fifty million miles.

Double stars are fascinating objects to study because of the variety of problems which they present. Their importance to astronomy, as indicated in this article, is enhanced by the knowledge they give us of stellar masses, luminosities, and dimensions. Without this source of information much of present day astrophysical science would rest on a flimsy foundation indeed. After the passage of two centuries the study of double stars continues to be an essential and vitalizing department of astronomy.

METEOR NEWS

By Peter M. Millman

National Research Council, Ottawa, Ontario

THE I.G.Y. VISUAL METEOR PROGRAMME—PROGRESS REPORT NO. 4.

This is the fourth tabulation of observations contributed by volunteers working on the special I.G.Y. programme of visual meteor observation. Previous summaries of observations were published in the JOURNAL, vol. 52, pp. 29 and 181, 1958; and vol. 53, p. 137, 1959. This current report completes the listing of all visual meteor observations made prior to the termination of the official 20-month I.G.Y. period. Subsequent observations, made in I.G.C. (1959) between February 1 and December 31, 1959, as a continuation of the I.G.Y. programme, will be reported later.

The observations are listed in Table I, the form of previous reports being used. The official station name is followed by the name of the group leader or organizer, or by the corresponding secretary in the case of a large group or club. The numbers of meteors listed are group totals, that is each meteor has been counted only once at any given station. The last column gives the average number of observers active at one time and includes the timekeeper. The four main divisions of the table include, in order, separate alphabetical listings of stations which appeared for the first time in each of the four reports respectively.

On the basis of periods worked the four largest contributors to this report have been G. Rippen at Des Plaines, Ill.; K. Delano at Baltimore, Md.; K. Komaki at Kanaya, Japan; and the Ottawa #1 station at meteor headquarters. A summary of the total numbers of meteors reported for the entire I.G.Y. period from June 1, 1957 to January 31, 1959 is given in Table II. Good progress is being made in transferring these observations to I.B.M. punched cards.

Sincere thanks go to all those who, through their interest and enthusiasm, have made possible the success of this programme.

TABLE I

SUMMARY OF OBSERVATIONS ON THE SPECIAL I.G.Y. VISUAL METEOR PROGRAMME RECEIVED AFTER OCTOBER 15, 1958

Location	Name	Nights	Ten-Minute Periods	Meteors Recorded	Average No. of Observers
Baltimore, Md.	K. Delano	47	479	739	1
Bexleyheath, England	K. Herbert	1	6	13	3
Boise, Idaho	J. Westby	2	40	409	3
Cape Elizabeth, Me.	R. Dole	14	159	302	1
Dallas, Tex. # 2	R. Althauser	3	75	417	7
Des Plaines, Ill.	G. Rippen	59	491	388	1
Dyer, Ind.	J. Berg	21	270	1143	1
Edmonton, Alta. #2	E. Milton	4	24	52	2
Elyria, Ohio	G. Diedrich	2	12	7	2
Fort Wayne, Ind. #2	D. Nelson	3	38	70	1
Greenwich, Conn.	J. Halloran	1	6	5	4
Hamilton, Ont. #1	J. A. Winger	2	26	76	4
Ions, Ohio	D. Hansen	5	110	122	1
Islip Terrace, N.Y.	W. Webster	28	132	98	1
Kingston, Jamaica	E. C. Melville	2	12	16	4
Lodi, Calif. #1	R. Birch	17	293	380	1
Lodi, Calif. #2	P. Sheehan	7	58	107	1
Lucerne, Switzerland #1	E. Roth	9	98	381	3
Minneapolis, Minn. #1	R. H. Scrimshaw	2	12	16	2
Montreal, Que.	Miss I. K. Williamson	4	53	96	5
Moose Jaw, Sask.	E. Freidin	5	32	112	6
New York, N.Y. #4	A. Pearlmutter	14	175	775	2
Omaha, Nebr.	C. Moroson	10	60	103	2
Ottawa, Ont. #1	Miss M. S. Burland	8	343	2108	5
Pittsburg, Pa.	W. A. Feibelman	21	180	269	1
Poughkeepsie, N.Y.	F. Greer	8	32	48	1
San Diego, Calif.	K. Yarab	2	16	16	1
Sandy Point, Me.	P. Miscall	2	12	10	1
Vancouver, B.C. #1	M. Taylor	2	13	132	4
Villa Rica, Ga.	W. F. Barber, Jr.	2	12	1	1
Western Springs, Ill.	R. Rexford	5	157	1056	5
West Nanticoke, (Berwick), Pa.	H. Humphries	4	36	2	1
Beacon Falls, Conn.	T. J. Bohuski	18	273	468	1
Broomall, Pa.	R. Field, Jr.	4	28	52	1
Centreville, Va.	M. E. Bishop	18	110	204	1
Cleveland, Ohio	K. Hicks	1	8	10	1
Danville, Ill.	M. Zillman	4	89	123	1
Houston, Tex. #1	D. Milon	9	87	191	2
Johannesburg, S. Africa	J. H. Botham	23	183	318	1
Kanaya, Japan #1	K. Komaki	44	370	483	1
Kibi, Japan	Mrs. F. Iwasaki	35	261	365	1
Kishiwada, Japan	Y. Maeda	14	182	360	—

TABLE I—*continued*

Location	Name	Nights	Ten-Minute Periods	Meteors Recorded	Average No. of Observers
Leavenworth, Wash.	S. Emig	6	99	371	2
Red Bluff, Calif.	F. Wyburn	19	182	95	1
Rio de Janeiro, Brazil #1	A. Fucs	2	14	13	1
Rio de Janeiro, Brazil #2	P. Silva	3	18	10	1
Rio de Janeiro, Brazil #3	H. Bucher	30	188	163	1
St. Paul, Minn.	R. A. Gorkin, Jr.	15	106	46	1
Sheffield, Ala.	R. May	25	211	275	1
Wakayama, Japan	A. Kamo	7	57	156	1
Washington, D.C.	W. L. Isherwood	2	12	24	8
Wellington, C.I., New Zealand	N. K. Keen	4	31	45	1
Davenport, Iowa	J. Schroeder	1	12	24	1
Houston, Tex. #2	J. Davis	3	22	11	1
Kanaya, Japan #2	K. Yamada	21	134	124	1
Maresca, Italy	G. Paolo	7	43	91	1
Port Credit, Ont. #1	J. Levis	1	3	5	1
St. Catharines, Ont.	G. Thomas	1	6	8	1
Schenectady, N.Y.	J. Keane	6	41	—	1
Seymour, Conn.	H. Kushpinsky	4	108	382	1
Winnipeg, Man. #1	J. Scatliff	2	11	26	1
Anapolis, Goias, Brazil	S. Dos Santos	4	54	27	1
Bellaria, Italy	F. Widener	2	27	98	1
Florence, Ala.	W. Smith	2	16	22	1
Gretna, La.	G. Fortmayer	14	72	48	1
Kanaya, Japan #3	Y. Hata	22	205	346	1
Latina, Italy	F. Gianpaolo	7	80	168	1
Lucerne, Switzerland #2	E. Roth	3	34	67	3
Northfield, Minn.	R. Althauser	2	37	304	2
Ottawa, Ont. #3	D. Fisher	1	7	37	1
St. Francis, Wis.	J. Jacoby	10	65	84	1
Westview, B.C.	R. K. Bradley	3	24	33	2
Winnipeg, Man. #2	D. Leckey	6	47	30	1
Winnipeg, Man. #3	N. B. McLean	1	15	28	1
Yamaguchi, Japan	A. Fujii	13	69	65	1
		TOTAL	7097	15151	

TABLE II

SUMMARY OF METEOR TOTALS

	Ten-Minute Periods	Meteors Recorded
Report 1	8,743	18,885
Report 2	7,722	14,997
Report 3	19,664	44,216
Report 4	7,097	15,151
GRAND TOTALS	43,226	93,249

NOTES FROM OBSERVATORIES

David Dunlap Observatory, Richmond Hill, Ontario

The members of the staff of the Observatory and the Department of Astronomy were hosts to the American Astronomical Society for its 103rd meeting held at the University of Toronto, August 30th to September 2nd. An account of the meeting has appeared in the December 1959 number of the Journal.

During the autumn the Observatory took part in the I.A.U. co-ordination programme of spectrographic observations of the eclipsing system VW Cephei. However, this programme as well as others was severely hampered by the unusually poor observing weather experienced here.

Work has begun on a modification of the photoelectric spectrophotometer attached to the 74-inch telescope which will extend the scanning of stellar spectra to stars of 11th magnitude or fainter.

A large portrait of the founder, the late Jessie Donalda Dunlap, which was painted in 1924 by Joshua Smith, R.B.A., has been presented to the Observatory by Mrs. D. M. Dunlap.

Ph.D. theses have been completed by Pierre Demarque ("The Structure of Sub-Dwarf Stars") and by Gustav Bakos ("Photometric and Spectrographic Investigations of Visual Binaries"), both of whom have now left to accept positions elsewhere. Other graduate students now enrolled are David Hogg, David Sher, Stephen Morris and Duncan McNeill. Enrolment in undergraduate astronomy courses at the University is heavy this session, totalling 200 and including five fourth year students in the graduating course in astronomy, who spend one day a week at the Observatory.

Members of the University teaching staffs in Astronomy and Zoology are collaborating in a series of colloquia entitled "The Universe and Life".

Dr. Searle conducted a colloquium on November 12th at the University of Rochester on "Element Abundances in Carbon Stars".

Recent visitors to the Observatory have included former Astronomer Royal Sir Harold Spencer Jones and Professor Robert Howard of the University of Massachusetts.

John F. Heard

VARIABLE STAR NOTES

By Margaret W. Mayall

The American Association of Variable Star Observers, Cambridge, Mass.

A total eclipse of the sun occurring near the time and place for our regular annual meeting sounded like a perfect arrangement, but the weatherman did not co-operate with us. The 48th Annual Meeting of the A.A.V.S.O. was held October 1–4, at Nahant, Massachusetts. The total eclipse scheduled for near sunrise on October 2 was rained out, but in spite of the rain, the sudden brightening of the sky at the end of totality was a startling and impressive sight.

The meeting was well attended, and the attractive country inn, the Edgehill, was filled to overflowing. At the business session, the 1958–1959 officers (see these Notes, vol. 53, p. 39), were all elected for another year. George Diedrich and Charles M. Good were re-elected for another 2 years on the Council, and William G. Cleaver and William H. Glenn were elected to fill the vacancies left by the 2 retiring members.

Fifteen applicants for membership in the Association were elected, as follows: Robert F. Farmer, *Texas*; Jay A. Frogel, *New York*; Anthony B. Hull, *Connecticut*; Rosalie Kunstadt, *New York*; Gary Larson, *Kansas*; Mrs. Nona B. Ledbetter, *Mississippi*; John W. Logan, *Massachusetts*; George F. McCahey, *Rhode Island*; C. G. Phillips, *Alberta, Canada*; Stephen C. Schiff, *Louisiana*; Robert N. Sealy, *Oregon*; Richard G. Skhal, *Arizona*; Norman C. R. Tabor, *Ohio*; Raymond Tollefson, *Alberta, Canada*; and Donald H. Zurstadt, *Indiana*.

Papers were presented on a variety of subjects. A solar and eclipse session was held at the Boston Museum of Science in collaboration with the Northeast Section of the Astronomical League. A brief history of the A.A.V.S.O. was given by Ralph N. Buckstaff, and one of the Solar Division and its current programmes, by Harry L. Bondy. Dr. Everett C. Rademacher showed coloured slides taken on the 1955 Bangkok eclipse expedition, and the Reverend Ronald Royer showed his coloured photographs of details of the Mount Wilson solar instruments, with Thomas Cragg at the controls.

An evening lecture was given by Dr. Dorrit Hoffleit entitled "Back to Variables". Other papers presented at the meetings included: *1959 Report of the Nova Search Division*, George Diedrich; *On the Observation of Shadow Bands*, Edgar M. Paulton; *Solar Eclipses*, Kenneth Weitzenhoffer; *Sun-spot Prediction Post Mortem*, Leith Holloway; *Another Star Illusion*, Patrick V. Rizzo; *Visual Meteor Observing*, Arthur Pearlmutter; *Fine Structure of the Solar Photosphere*, Constantin Chassapis; *Miscellany*, Margaret W. Mayall; *Report on A.A.V.S.O. at Denver Convention*, Clinton B. Ford; *Folding Astronomy*, R. Newton Mayall, and *Some Remarks about the A.A.V.S.O. Solar Division*, Harry L. Bondy. Summaries of all these papers are published in the A.A.V.S.O. Abstracts, edited by R. Newton Mayall.

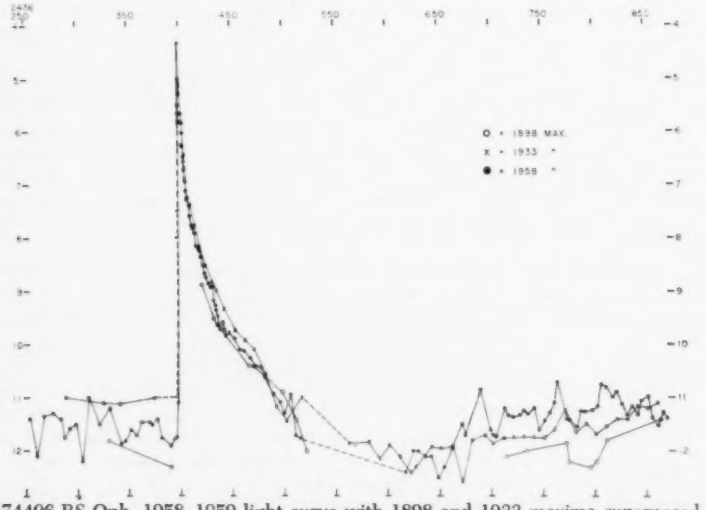
The Annual Report of the Director showed that 52,589 observations of variable

stars were received during the year, plus a total of 24 hours and 30 minutes of constant observation of flare stars and other rapidly changing variables. These observations were received from 184 observers in 18 countries. The observer who made the greatest number of observations during the year was Reginald P. de Kock with 6,582, and second was Cyrus F. Fernald with 4,758. Both of these men were also members of the "Inner Sanctum" group, who make observations of stars fainter than 13.8 magnitude. Ten other observers made more than 1,000 each. They are: Edward Oravec, 4,174; Thomas Cragg, 2,704; Curtis Anderson, 2,504; Leif Robinson, 2,133; Robert Adams, 1,987; Clinton Ford, 1,840; Ferdinand Hartmann, 1,756; Leslie Peltier, 1,310; Robert Allin, 1,071; and Domingo Taboada, 1,002.

The fainter variables are receiving more attention each year, as more of our observers who have large telescopes make special efforts to observe as faint as possible. This year 29 observers were "Inner Sanctum" members. Our great problem now is to get magnitudes of fainter comparison stars, to take advantage of the large instruments available. Among our members we now find Claude Carpenter with an 18″ reflector, J. Russell Smith with a 16″ reflector, Leslie Peltier with a 12″ Clark refractor, and many others with 12″ reflectors. The Milwaukee Astronomical Society has a 13″ reflector, and the Berlin amateurs have a 12″ refractor, used by Werner Braune and Reinhard Rudolph.

In addition to these telescopes in regular use on visual programmes, we have had observations made with the Mount Wilson 100″ reflector by Allan Sandage; the Mount Wilson 60″ reflector by Thomas Cragg and George Wallerstein; the Yerkes 40″ reflector by Dale Cruikshank and James Breckinridge; and the Lick 36″ reflector by George Herbig.

The Photoelectric Photometer division under the chairmanship of Lewis Boss has been very active. John Ruiz continues to do fine work on some of the international



174*406* RS Oph. 1958–1959 light curve with 1898 and 1933 maxima superposed.

programmes of co-operation on observations of very short period variables. Domingo Taboada, in Mexico, has joined some of these observing programmes. Donald Engelkemeir has made many hours of runs on confirmed and suspected flare stars. He was fortunate enough to catch a beautiful flare of AD Leonis. Several other members have photoelectric photometers in the process of construction and testing.

RS Ophiuchi, Nova 1898, 1933 and 1958. A comparison of the three maxima of RS Ophiuchi 174406 shows quite conclusively that the star is well named a recurrent or repeating nova. The accompanying light curve is a plot of mean values of A.A.V.S.O. observations of the 1958 maximum, and includes the observations of the New Zealand observers through December 31, 1958 (J.D. 2436569). The mean light curve of the 1933 maximum is superposed by matching the observed times of maxima of the 2 curves. The 1898 maximum is the least well observed of the three, but the few observations we have fall on the other curves in such a way as to leave little doubt that the true maximum must have been about 5th magnitude. The agreement of the 1898 observations would look even better on this plot if the interpolated time of maximum were shifted about 5 to 10 days earlier, thus moving the points to the right on the diagram.

The following observations made immediately before and during the 1958 maximum show how well observed it was:

J.D.	*Mag.*	*Observer*	*J.D.*	*Mag.*	*Observer*
2436394.3	12.2	de Kock	2436398.65	5.5	Fernald
.6	11.4	Fernald	.66	5.5	Fernald
.9	11.7	Jones (N.Z.)	.67	5.6	Ford
396.3	11.9	Smits	.67	5.3	Ford
.6	11.4	Fernald	.9	5.0	Wallerstein
.7	11.9	Peltier	399.7	5.5?	Cragg
397.7	11.1	Peltier	.7	5.15	Sandage
398.57	6.2	Fernald	.8	5.25	Sandage
.59	6.0	Fernald	400.3	6.0	de Kock
.60	5.0	Rosebrugh	.4	5.2	Lacchini
.61	5.7	Fernald	.4	5.5	Lacchini
.62	5.6	Fernald	.6	5.69	10 observers
.64	5.4	Fernald			

Errata: JOURNAL, Vol. 53, No. 4, p. 194. The co-ordinates on the light curve in figure 2 were given incorrectly. On the top line, the Julian Days should read: 3000, 3500, 4000; and second line: 4000, 4500, 5000.

A.A.V.S.O. Nova Search Report (From George Diedrich, Chairman): Observations of Nova Search Areas were made by the following 22 observers for a total of 764 area-nights. Each name is followed by the number of observations made in July and then August, 1959: Robert Bishop–28, 21; James Breckinridge–10, 5; Buddy Brown –25, 35; F. J. DeKinder–16, 12; DeLorne Diedrich–10, 7; George Diedrich–14, 12; C. L. Drolet–24, 17; Paul Evans–46, 49; Kenneth Fuller–15, 41; Geoffrey Gaherty, Jr.–14, 4; William Isherwood, Jr.–11, 21; John Josephson–3, 0; Howard Nightingale –27, 49; D. W. Orchiston–94, 34; Tom Osypowski–1, 1; Robert Price–4, 0; G. A. Spencer–8, 8; F. Traynor–4, 5; G. Vodrazka–0, 10; G. Wedge–13, 15; I. K. Williamson–17, 9; D. Zorgo–25, 0.

It is of interest to note that over 40 per cent. of the above observations were made by the 6 observers from New South Wales, Australia! We need more observers. Have you any friends you could interest in Nova Searching? Have them drop a card to the Chairman at 653 Weller Road, Elyria, Ohio, U.S.A.

Observations received during September and October 1959: A total of 10,435 observations was received for the two months: 4,888 from 93 observers in September, and 5,547 from 88 observers in October.

Observer	Sept. Var.	Sept. Ests.	October Var.	October Ests.	Observer	Sept. Var.	Sept. Ests.	October Var.	October Ests.
*Adams, R. M.	66	130	83	182	Kindt, O. H.	14	15	..	..
Allin, R.	24	29	9	9	Knowles, J. H.	32	71	23	51
Anderson, C. E.	34	46	80	102	Kopp, R.	4	12	..	..
Anderson, D. V.	..	..	1		Lacchini, G. B.	36	74	41	123
Anderson, E.	..	..	6	101	Lattey, M.	5	11	7	7
Andrassy, P.	2	7	..	..	Ledbetter, N.	14	14	24	28
Apolito, T.	1	4	..	..	Lowder, W. M.	32	61	58	84
Bales, R. M.	11	13	..	..	*Mayall, M. W.	7	11	..	..
Ball, A. R.	..	..	12	16	Mayall, R. N.	2	5	..	..
Bartha, L.	4	.4	..	..	Maran, S. P.	..	..	7	7
Beidler, H. B.	16	16	20	20	McFarland, T.	6	6	13	24
Berg, J.	12	16	15	29	*McPherson, C. A.	39	57	33	38
*Berg, R.	13	17	..	..	Miller, R. W.	..	..	5	5
Bieda, S. W.	..	..	4	7	Miller, W. A.	11	12	7	7
Brady, R. F.	7	8	9	17	Montague, A. C.	16	17	22	28
Braune, W.	..	..	33	324	Morgan, F. P.	..	..	13	30
Brechlin, D.	..	..	2	2	Muniz B., L.	12	14	6	10
Breckinridge, J.	2	2	32	40	Nightingale, H. C.	3	14	3	29
Buckstaff, R. N.	..	..	24	89	Olsen, N.	1	1	..	..
Carlisle, J. H.	1	3	..	..	Oravec, E. G.	172	334	142	317
Conklin, J. H.	..	..	3	3	Orchiston, D. W.	13	121	37	173
Constanten, T. C.	6	20	17	50	Parker, P. O.	45	50	..	..
Cragg, T. A.	127	133	363	375	Pearcy, R. E.	3	3	1	1
*Darsenius, G.	11	73	12	39	Peltier, L. C.	26	187	25	87
de Kock, R. P.	134	710	144	572	Peters, P.	8	10	..	..
Diedrich, DeL.	2	4	4	6	Pinter, —.	3	3	..	..
Diedrich, G.	2	6	7	13	Price, R. T.	11	26	12	27
Dudley, R. R.	16	19	11	12	Quester, W.	..	..	11	95
Ellerbe, J. E.	5	9	2	3	Renner, C. J.	..	..	103	103
*Engelkemeir, D.	9	11	8	21	Ries, J.	22	37	8	13
Erpenstein, O. M.	6	8	11	26	Rizzo, P. V.	29	29	18	25
Evans, P.	13	99	11	103	Robinson, J. C.	28	39	..	..
Fejes, —.	5	9	..	..	Robinson, L. J.	2	2	12	218
*Fernald, C. F.	199	491	237	615	Roderick, T.	3	3	..	..
Fernald, E.	1	1	..	..	Rosebrugh, D. W.	11	73	13	104
*Fletcher, D. M.	..	..	7	14	Rosenfield, C. C.	14	14	17	22
Ford, C. B.	104	108	128	132	Rosenfield, D. A.	10	13	8	16
Fuller, K.	77	79	55	56	Royer, R.	..	..	3	3
Gauser, —.	1	1	..	..	Rudolph, R.	..	..	48	222
Giffen, C. H.	130	438	33	33	Segers, C. L.	..	..	48	133
Glagola, G.	..	..	6	6	Sharpless, A. P.	9	9	19	19
Godfrey, N. B.	..	..	20	20	Shinkfield, R. C.	10	28	15	18
Grossenbacher, R.	11	11	36	41	Smith, G. W.	11	11	7	7
Hartmann, F.	153	171	145	149	*Smith, J. R.	1*	..	29	34
Hiett, L.	9	26	10	22	Smits, P.	77	106	46	46
Hull, A.	..	..	4	5	Solomon, L.	12	14	7	7
Humphrey, D. R.	1	7	2	13	Staer, R. R.	1	4	1	18
Judd, D.	..	..	8	8	Stanton, R.	11	17	9	16
Kaminski, W. J.	37	73	4	4	Szekely, —.	3	3	..	..
Kammel, J.	1	4	..	..	Taboada, D.	34	34	..	..
*Kelly, F. J.	18	19	17	20	Templeton, J. H.	5	5	5	6

Observer	Sept.		October		Observer	Sept.		October	
	Var.	Ests.	Var.	Ests.		Var.	Ests.	Var.	Ests.
Thaly, K.	6	38	..	..	Weaverling, W.	15	28	..	..
Thurn, V. W.	..	..	6	6	*Welker, F.	..	..	7	7
Traynor, F.	5	11	4	10	Wilford, F.	8	9	..	..
Tsai, C. H.	46	101	..	..	Williamson, L. J. A.	3	24	..	..
Ullmann, A.	1	1	..	..	Williamson, P. L.	2	10	..	2
Valdez, M.	4	5	1	1	Yakish, S.	..	..	2	100
Van Til, A.	..	..	2	2	Yamada, T.	62	129	32	..
van Zyl, L. L.	24	133	..	..	Zellner, B.	4	4	..	..
Vodrazka, G.	..	..	1	9					

*In addition to the regular estimates listed above, some observers made continuous runs of observations on special variables, as follows: R. M. Adams, 30^m V Sge; R. Berg, 45^m AE Aqr, 10^m V Sge; G. Darsenius, 9^m DO Cep, 67^m EV Lac; D. Engelkemeir, 54^m V Sge, $3^h\ 30^m$ PEP+51°2402; C. F. Fernald, 12^m V Sge; D. M. Fletcher, 13^m V Sge; F. J. Kelly, 15^m V Sge; M. W. Mayall, 19^m V Sge; C. A. McPherson, 1^h V Sge; J. R. Smith, $1^h\ 40^m$ V Sge; F. Welker, 5^m V Sge.

REVIEW OF PUBLICATIONS

Astronomischer Jahresbericht edited by K. Heinemann, F. Henn, W. Lohman. Volume 57, The Literature of the Year 1957. Astronomischen Rechen-Institut in Heidelberg. Pages xxxviii + 525; 5⅝ × 8⅜ in. (paper). Berlin, Walter de Gruyter & Co., 1959. Price 60 DM.

As a summary of all astronomical research literature, this publication has been invaluable for many years. And its value increases as the number of astronomical periodicals and the pages of astronomical literature grow with time. The editors are to be commended for this colossal task, accomplished so efficiently. As an example of how their work has increased with the years, we note that volume 39, containing literature of the year 1937, had 100 sections, totalling 329 pages. Now volume 57, the literature of the year 1957 has 134 different sections, totalling 525 pages. These sections cover all branches of astronomy, including astronautics and rockets. In addition to them, a name index of authors of papers provides a handy guide to the names of research astronomers, and the 26 pages devoted to listing all astronomical periodicals and observatory and institute publications give the reader a global idea of where astronomy is being done.

Just because this volume appears every year we should not take it for granted, but on the contrary should always be grateful for its reappearance. The only improvement one could suggest is that the publication lag time be reduced, a goal toward which the editors have always worked.

HELEN SAWYER HOGG

CORRECTION TO DECEMBER 1959 JOURNAL

On page 263 of the December issue of the JOURNAL in the article entitled "The Figure of the Earth" by R. K. C. Johns, the figure for the flattening of the earth should read 298.3 (instead of 281.3).

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